Alumina Particle Velocity and Temperature in a Solid Rocket Plume

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As part of a study of the smoke characteristics of solid propellant rockets, a mathematical model has been developed for prediction of the velocity and temperature of alumina particles on the axis of the plume. The model assumes that the particles are spherical, and sufficiently diluted in the plume so that there is no flow interference with the main gas stream nor interaction among the particles. Momentum and thermal effects of shocks in the plume, and radiation were not included in the analysis. The calculated results for 5, 10, and 20μ radius particles in the exhaust from an ammonium perchlorate-rubber composite propellant are presented in Figs. 1 and 2.

Previous papers by Kliegel,[12] Hoglund,[13] and Carlson and Hoglund[14] were concerned with particle velocity and temperatures in the rocket nozzle. The present paper is an extension of this work into the external plume itself.

Since the gas density in a rocket exhaust is very low compared with the density of an alumina particle, the only forces on the particle that need to be considered are body and surface drag F_D, transient flowfield response—the Basset history term F_H, and external potential fields F_e, e.g., gravity. The forces due to the flow pressure gradient and the so-called "added-mass' are negligible.[1]

$$\pi/6D_p^3\rho_p \mathrm{d}V_p/\mathrm{d}t = F_D + F_H + F_e \tag{1}$$

where D_p, ρ_p, and V_p are the diameter, density, and velocity of the particle, respectively. The alumina particle velocity relative to the gas is generally outside the regime of Stokes flow, and therefore experimental drag coefficients must be used to compute F_D, as defined by the equation

$$F_D = -\tfrac{1}{2} C_D \rho_g (\pi D_p^2/4)\, |V_p - V_g|\, (V_p - V_g) \tag{2}$$

The subscript g refers to the gas and C_D is the drag coefficient.

The correlation of Beard and Pruppacher[2] for drag coefficients in incompressible continuum flow C'_D was used

$$0.2 \le N_{\mathrm{Re}} \le 2 \quad C'_D = (24/N_{\mathrm{Re}})(1 + 0.1 N_{\mathrm{Re}}^{0.99}) \tag{2a}$$

$$2 \le N_{\mathrm{Re}} \le 21 \quad C'_D = (24/N_{\mathrm{Re}})(1 + 0.11 N_{\mathrm{Re}}^{0.81}) \tag{2b}$$

$$21 \le N_{\mathrm{Re}} \le 200 \quad C'_D = (24/N_{\mathrm{Re}})(1 + 0.189 N_{\mathrm{Re}}^{0.63}) \tag{2c}$$

The Reynolds number N_{Re} is based on the absolute value of the velocity difference between the particle and the gas stream, and the physical properties of the gas calculated at the temperature of the bulk gas. However, since the flow deviates from the continuum and enters the slip and transition flow regimes between continuum and free molecular flow, corrections to the Beard and Pruppacher C'_D values must be made. Zarin[3] conducted wind-tunnel tests in the flow regimes of interest, and reported that the correction for slip and rarefaction could be best predicted (although a few per cent low) by the algebraic expression developed by Crowe.[4]

$$C_D = (C'_D - 2)\exp[-3.07\gamma^{1/2}(M/N_{\mathrm{Re}})g(N_{\mathrm{Re}})] + (h(M)/\gamma^{1/2}M)\exp(-N_{\mathrm{Re}}/2M) + 2 \tag{3}$$

with M the Mach number and γ the ratio of specific heats of the gas.

$$\log_{10} g(N_{\mathrm{Re}}) = 1.25[1 + \tanh(0.77\log_{10} N_{\mathrm{Re}} - 1.92)]$$

and

$$h(M) = [2.3 + 1 + 7(T_p/T_g)^{1/2}] - 2.3\tanh(1.17\log_{10} M)$$

where T is the absolute temperature, °R. For the flow regimes examined here, the term in Eq. (3) incorporating $h(M)$ was found to be negligible.

The Basset history term was originally derived for creeping flow. Odar[5] found that a correction coefficient C_H must be included for high acceleration fields

$$F_H = -3/2 C_H D_p^2 (\pi\mu_g\rho_g)^{1/2} \int_0^t (\mathrm{d}V_p/\mathrm{d}t' - \mathrm{d}V_g/\mathrm{d}t')\mathrm{d}t'(t-t')^{-1/2} \tag{4}$$

with μ_g the absolute gas viscosity. For large ratios of convective acceleration to local acceleration, Odar found that C_H approached an asymptotic value of 2.88. Lewis and Gauvin[6] reported that this value of C_H satisfactorily correlated their data for the motion of glass spheres, 30-140μ in diameter, entrained in a free argon plasmajet. This value of C_H was used in the present analysis.

The factors of rarefaction and slip-flow influence convective heat transfer as well. Using the Drake et al.[7,8] correlation for Nusselt number in continuum flow Nu_0 corrected for rarefaction and slip flow, Nu

$$Nu_0 = 2 + 0.459 N_{\mathrm{Re}}^{0.55} N_{\mathrm{Pr}}^{0.33} \tag{5}$$

$$Nu = Nu_0/[1 + 3.42[M/N_{\mathrm{Re}}N_{\mathrm{Pr}})]Nu_0] \tag{6}$$

The change in particle temperature with time is expressed by

$$\mathrm{d}T_p/\mathrm{d}t = -hA(T_g - T_{\mathrm{ad}})/(C_p\rho_p v) \tag{7}$$

Nu and Nu_0 are correlated with N_{Re} and N_{Pr}, the Prandtl Number, calculated from $|V_p - V_g|$ and the freestream gas properties at T_g, the freestream gas temperature, and the gas thermal conductivity calculated at the gas recovery temperature, T_{ad}. A and v are the surface area and volume of the sphere, respectively, and h the heat-transfer coefficient.

No data are available for the recovery factor of spheres traveling at subsonic relative velocities, and $N_{\mathrm{Re}} < 75$, the condition of interest here. Drake and Backer[9] report data for recovery in supersonic flow in the transition regime with recovery being 0.9 for values of $\sqrt{N_{\mathrm{Re}}}/M > 5$ in the range of $\sqrt{N_{\mathrm{Re}}}/M$ from 1.5 to 9. Values of $\sqrt{N_{\mathrm{Re}}}/M$ in the present study were about 13. A recovery factor of 0.9 was used.

Using the AeroChem plume computer program,[10] gas velocity, temperature, and composition vs axial distance from the nozzle exit were calculated for the case of an 86% ammonium perchlorate-hydroxy terminated butadiene propellant burning at 1000 psia expanding through an optimum nozzle into 14.7 psia still air. The equations of motion and heat transfer for the alumina particles in the plume, described above, were integrated numerically using the Merson version of the Runge-Kutta predictor-corrector routine.[11]

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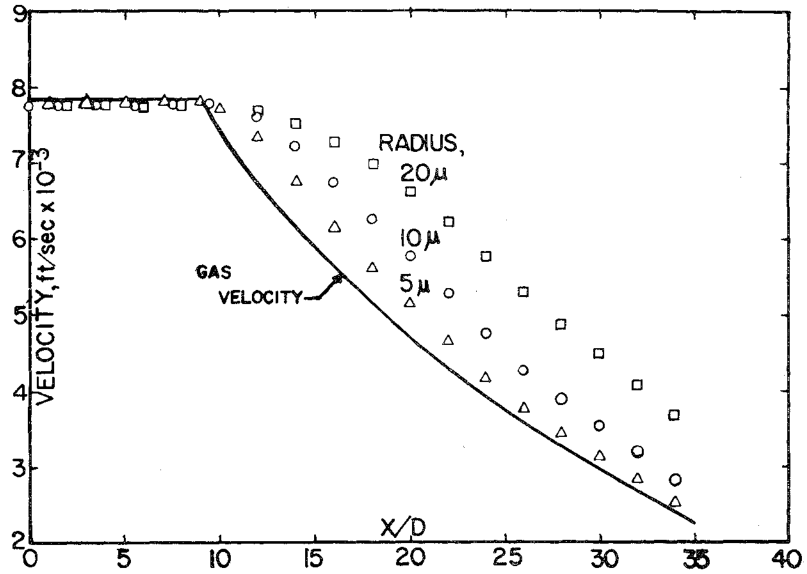

Fig. 1 Calculated velocity vs dimensionless distance in nozzle diameters.

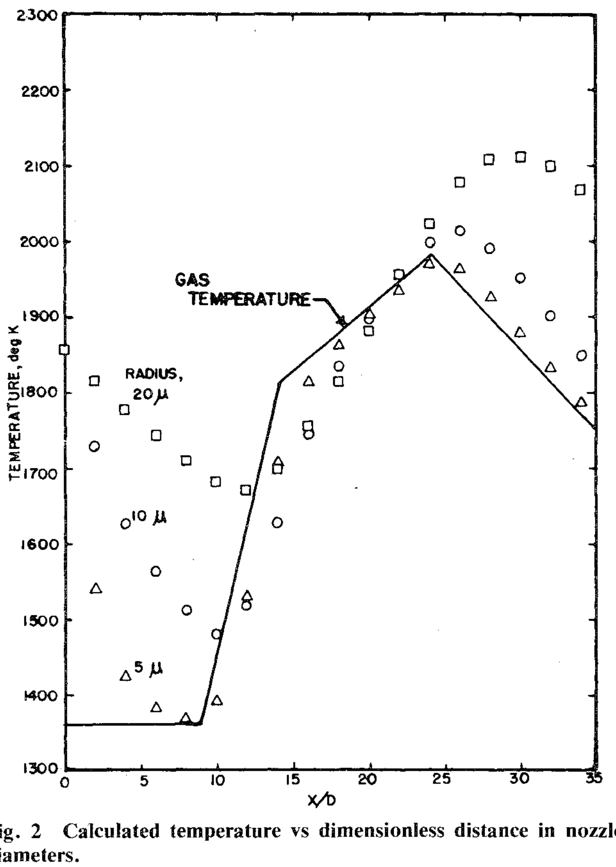

Fig. 2 Calculated temperature vs dimensionless distance in nozzle diameters.

The particle velocity and temperature at the nozzle exit were arbitrarily chosen at 100 fps less and 500K greater than the gas velocity and temperature, respectively.

The calculated values of the axial gas temperature given in Fig. 2 show an increase after the end of the potential flow core. This increase is due to after burning of the fuel rich exhaust with the admixed air. The break-points in the plot of temperature vs X/D are not physically significant, but result from linearization of the temperatures in the intervals of X/D shown.

As may be seen in Fig. 1, in the potential core there is only a small increase in the particle velocity. Downstream of the core, the gas velocity decreases rapidly and the velocity difference between the gas and the particle increases to 635-1930 fps for the 5 and 20μ particles, respectively. Even after 34 nozzle-diameters, the maximum distance for which calculations were made, the velocity differences are still large (145 and 1305 fps for the 5 and 20μ particles).

Significant segregation of particles by size must also take place with distance. The Basset term was an important factor in the calculation of the velocity of the particle only where the difference in deceleration of the gas and particle was the greatest—from the end of the potential core (8.75 diam) out to about 10 diameters. The range of Basset drag was a maximum of -60 to $+43\%$, decreasing to 3.5% of the body drag for the 20μ particle and -6 to $+18\%$, decreasing to 1% of the body drag for the 5μ particle. In addition, for the 5μ particle the Basset term averaged about -2.5% of the body drag in the potential core.

As shown in Fig. 2 for the gas and particle temperature variations, the 5μ particle rapidly approaches the temperature of the gas and follows it closely thereafter. This is not the case for the larger particles where a significant "undershoot" and "overshoot" of the gas temperature are predicted. The predicted particle velocities and temperatures should have important implications for the prediction of plume signatures of solid rockets.

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Correlation of Transition Reynolds Number with Aerodynamic Noise Levels in a Wind Tunnel at Mach Numbers 2.0-3.0

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THE correlation of model transition Reynolds number in the aeroacoustic environment of a wind tunnel with that which would occur in free flight is a long-sought and elusive goal. The reason is that transition data in wind tunnels have been enshrouded with effects associated with the test facility and not the model. An important advancement to the understanding of facility environmental influence on transition was the work of Pate and Schueler,[1] which supposed a strong effect of radiated aerodynamic noise from the tunnel wall boundary layer in supersonic and hypersonic test facilities. Verification of the existence of this effect was obtained by shielding a model from the tunnel walls with a shroud.

Pate and Schueler were able to correlate transition Reynolds number data from 10 different wind tunnels using an empirical expression containing the tunnel wall skin friction coefficient (C_f) and displacement thickness (δ^*) to characterize the noise. Because the amount of noise reaching the model depends upon radiation laws, the correlation necessarily included a tunnel size parameter, which was based on the test section perimeter (c). Different empirical constants were found when cone and planar model geometries were considered.

Their empirical correlation was demonstrated to hold over a Mach number range of 3.0-8.0 for tunnel sizes ranging from 1.0 ft square to 16 ft square. In the largest tunnel, the AEDC 16 ft supersonic propulsion wind tunnel (PWT16S), data were acquired on a 12-in.-diam hollow cylinder at Mach numbers 2.0, 2.5, and 3.0. The test-section boundary-layer properties were measured at a reference location 69.9 ft from the nozzle throat, which was behind the model and near the middle of the test section. The data at Mach numbers 2.0 and 2.5, although documented in Ref. 1, were not used in the correlation because the transmissibility of turbulence from the stilling chamber through a supersonic nozzle increases greatly as the Mach number is decreased below 3.0.

Subsequent to these tests, additional data have been acquired by PWT personnel in tunnel 16S, including additional documentation of wall boundary-layer skin friction and thickness, direct measurements of noise level in the test section, and turbulence level in the stilling chamber. These data were acquired at various unit Reynolds numbers over the Mach number range of 1.67-3.0. The stilling chamber turbulence level (axial component, $\bar{u}'/U \times 100$) was found to be quite low, 0.2% maximum. Considering the level of turbulence entering the test section after the nozzle contraction, it was concluded from the test section noise measurements that the disturbances in the test section were predominantly acoustic (turbulent boundary-layer radiation) for all Mach numbers from 1.67 to 3.0.

The noise measurements were made using a flush-mounted microphone on the surface of a 10-deg cone. This is the same cone that was used in a transition correlation study which has been carried out primarily in transonic tunnels.[2] The cone model was mounted so that location of the microphone was at a tunnel axial station 58 ft from the throat. The cone had insufficient length to obtain complete transition on the cone within the Reynolds number capability of the tunnel, hence all of the noise measurements were made with the microphone under a laminar boundary layer. The cone tests provided noise measurements that should be fairly representative of the freestream noise level, free from local turbulence, which would have been present from a measurement under a turbulent or transitional boundary layer.

Recent boundary-layer measurements made on different walls of 16S at different tunnel axial stations[3] showed that the variations of boundary-layer characteristics with Mach number and Reynolds number in 16S can be correlated by the method of Winter and Gaudet.[4] Use of this correlation was the basis for extrapolating all of the recent boundary-layer data to the Pate and Schueler reference station of 69.9 ft. The extrapolated boundary-layer characteristics were correlated (in particular, the skin friction) with the hollow cylinder transition data. It was found that the Mach 2.0 and 2.5 data also could be correlated, using Pate and Schueler's method.

Furthermore, after deriving a correlation between the wall boundary-layer properties and the noise data, a direct correlation was found to exist between the Ref. 1 transition data and the noise levels measured on the cone. The purpose of this Note is to present these correlations.

The transition Reynolds number data to be used herein were interpolated at four unit Reynolds numbers from those measured on a hollow cylinder with a leading edge lip bluntness of 0.0015 in. Table 1 gives the values of transition Reynolds numbers obtained.

These data (Re_T) represent the end of transition length Reynolds number based on the peak in pressure measured by four pitot probes traversing axially along the exterior surface of the cylinder, as described in Ref 1. An average of the four simultaneous readings circumferentially spaced 90 deg apart was taken at each test condition to eliminate any flow angularity error. Correlation of the data given in Table 1 with the correlation from other tunnels given in Ref. 1 is shown in Fig. 1. These data, which are correlated using information

Table 1 Transition Reynolds numbers

M_∞	$Re/ft \times 10^{-6}$	$Re_T \times 10^{-6}$
2.0	1.2	3.60
	1.5	4.06
2.5	0.6	2.40
	0.9	2.79
	1.2	3.09
	1.5	3.41
3.0	0.6	2.11
	0.9	2.45
	1.2	2.69

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